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The importance of attribute interactions in conjoint choice design and modeling

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Abstract

Within the context of choice experimental designs, most authors have proposed designs for the multinomial logit model under the assumption that only the main effects matter. Very little attention has been paid to designs for the attribute interaction models. In this paper, we present Bayesian *D*-optimal interaction-effects designs for the multinomial logit models and compare their predictive performances with those of main-effects designs. We show that in situations where a researcher is not sure whether or not the attribute interaction effects are present, incorporating interaction effects into both design stage and model estimation stage is most robust against misspecification of the underlying model for making precise predictions.

Keywords: choice experimental designs, main effects, interaction effects, Bayesian *D*-optimal design

1. Introduction

Conjoint choice experiments have become increasingly popular as a major set of techniques for studying consumer choice behavior. These experiments enable researchers to model choices in an explicit competitive context, thus realistically emulating market decisions (Arora and Huber 2001; Carson et al. 1994; Zwerina et al. 1996). Choice data instead of rating and ranking data are collected in conjoint choice experiments. The respondents or consumers are asked to express their preferences by choosing products rather than by ranking or rating them. A popular model for analyzing choice data is the multinomial logit (MNL) model (McFadden 1974). The conjoint choice designs presented in this work are based on this model.

A serious difficulty in the construction of an efficient choice design for the MNL model is that it requires knowledge of the values of the parameters (Atkinson and Donev 1992; Atkinson and Haines 1996; Sandor and Wedel 2001). This is different from optimal designs for linear regression models where the information provided by the design does not depend on the model parameters. Kessels et al. (2005) summarize three approaches for coping with the problem of the optimal designs' dependence on the unknown parameters. The first approach is to use zero prior parameter values. In that approach, it is implicitly assumed that the respondents have no particular preference of one alternative over another. This assumption simplifies the design problem considerably as the nonlinear experimental design problem is then simplified to the linear experimental design problem. The second approach, referred to as the local optimal design approach, is to use nonzero prior values, where a prior point estimate β_0 is used to construct the design. This approach was adopted by Huber and Zwerina (1996), who showed that if β_0 is reasonably close to the unknown true value, the resulting locally optimal design is more efficient than that obtained by using zero prior values. The third approach is the Bayesian optimal design approach introduced in the marketing literature by Sandor and Wedel (2001), who extended the work of Huber and Zwerina (1996). The Bayesian approach takes into account the uncertainty about the assumed prior parameter value β_0. Sandor and Wedel (2001) describe different situations in which the Bayesian designs provide higher efficiencies than the corresponding locally optimal designs. In the present paper, the Bayesian optimal design approach is applied.

In most of the literature on the optimal design of conjoint choice experiments, researchers focus on optimal main-effects designs for the MNL logit model, and neglect interactions between attributes (e.g., Bunch et al. 1996; Kessels et al. 2005; Sandor and Wedel 2001). However, in design of experiments, the identification and estimation of interactions between experimental factors is generally regarded as very important (Blomkvist et al. 2000). One of the advantages of the optimal choice design approach proposed by Zwerina et al. (1996) is that allows the incorporation of attribute interactions in the design stage. The approach these authors adopt is a locally optimal design approach, in which zero prior values for the interaction effects are used. Table 1 gives an overview of the optimal conjoint choice design papers that have recently appeared. We indicated whether these papers focus on main effects designs or take interactions into account.

In some practical conjoint choice design problems, marketing researchers have prior beliefs about whether or not interaction effects are present. In those cases, there is relatively little uncertainty about the precise nature of the model to be estimated, and, consequently, about the model for which to compute an optimal experimental design. There are, however, situations in which the researcher is unsure about the presence or absence of interaction effects. The purpose of this article is to quantify the impact of ignoring possibly important interactions in both the modeling stage and the design stage. Unlike in some experimental design problems outside the conjoint experiment context where designs for one model don't even allow the estimation of a competing model, optimal choice designs constructed for a main-effects model usually allow the estimation of a model involving interactions. Therefore, the design choice for conjoint choice experiments is less critical than for other types of experiments. In this paper, we investigate how important it is to take into account the interaction effects in the design and analysis stages. The purpose is to provide the reader with a model-robust strategy to conduct a conjoint choice experiment and to analyze the resulting data that guarantees precise predictions no matter what the ultimate model turns out to be.

The article will be based on two competing models, namely a main-effects MNL model and an interaction-effects MNL model. We assume that the researcher has the choice between constructing a design for either the main-effects model or the interaction-effects model when constructing an optimal design for the experiment, and that, when analyzing the data from the experiment, the researcher can also choose either to include interactions in the model or not. A main-effects design, constructed assuming a model with only main effects was the true one, and an interaction-effects design, constructed under the assumption that an interaction effect was active, are compared. To compute the Bayesian optimal designs for both models, we used the Bayesian modified Fedorov choice algorithm proposed by Kessels et al. (2005). For each design, both the main-effects model and the interaction-effects model were fitted. So, four different combinations of design and analysis strategy are compared here to study how the attribute interactions influence the predictive performance of the chosen model and design, and which combination of design and estimation approach is most robust against the misspecification of the underlying model.

Table 1

Main Contributions of Previous Studies

Authors	Type of design	Main focus of their paper	Approach
Bunch et al. (1996)	Main-effects design	• Comparing a variety of different designs for choice experiments	Locally *D*-optimal
Sandor and Wedel (2001)	Main-effects design	• Taking into account prior information about the parameters and the associated uncertainty • Proposing a way to elicit prior information from managers	Bayesian *D*-optimal
Kessels et al. (2005)	Main-effects design	• Taking into account the uncertainty about the assumed parameter values • Comparing designs produced by different optimality criteria	Bayesian *D*-, *A*-, *G*- and *V*-optimal
Huber and Zwerina (1996)	Main-effects design and interaction-effects design	• Including nonzero prior parameter values in the design stage • Studying the importance of utility balance in efficient choice designs	Locally *D*-optimal
Zwerina et al. (1996)	Main-effects design and interaction-effects design	• Proposing a general strategy for the computerized construction of efficient choice designs	Locally *D*-optimal

The remainder of this paper is constructed as follows: the next section provides a brief review of the main ideas in generating efficient choice designs. Section 3 presents the comparison of the interaction-effects design and the main-effects design. In Section 4, the predictive performances of the two types of designs and two types of analyses are compared. Section 5 contains the final conclusion of this study.

2. Bayesian Conjoint Designs

2.1 The multinomial logit model

In choice based conjoint experiments, R respondents are presented with a number of choice sets, each set consisting of several alternatives. The respondents are requested to indicate which alternative they prefer in each choice set. The model most often used to analyze the data from such experiments is the well-known multinomial logit model (McFadden 1974) with the assumption that the coefficient β is the same across respondents. The random utility for an alternative i in choice set n for a given person r is modeled as

(1) $$u_{inr} = x'_{in}\beta + \varepsilon_{inr},$$

where x_{inr} is a p-dimensional vector characterizing the attributes of alternative i in choice set n, β is a p-dimensional parameter vector reflecting the weights of these attributes. Finally, ε_{inr} is an i.i.d. error term with an extreme value distribution. If the number of alternatives within a choice set is denoted by I, the probability that alternative i is chosen from choice set n is

(2) $$p_{in} = \frac{\exp(x'_{in}\beta)}{\sum_{k=1}^{I}\exp(x'_{kn}\beta)}.$$

If we assume that all R respondents receive the same choice sets and denote the number of these choice sets by N, the log-likelihood function for the MNL model can be written as

(3) $$\ln\{L(\beta)\} = \sum_{r=1}^{R}\sum_{n=1}^{N}\sum_{i=1}^{I} y_{inr}\ln\{p_{in}\},$$

where y_{inr} is a binary variable that equals one if respondent r chooses alternative i in choice set n, and zero otherwise. The maximum likelihood estimate $\hat{\beta}$ for the parameter vector β is the set of values that maximizes the log-likelihood function.

2.2 Design Efficiency Criterion

The D-criterion is undoubtedly the most frequently used criterion to design choice experiments. One of the advantages of this criterion over other optimality criteria is that it is invariant to the scale or coding of the attributes. That is, the relative efficiency of the designs does not change when different codings of the attributes are used (Goos 2002). Another motivation for using the D-optimality criterion was given by Kessels et al. (2005). They show that D- and A-optimal designs are nearly as good as the G- and V-optimal designs in terms of prediction quality but much faster to compute compared to G- and V-optimal designs. Also, compared to the A-optimality criterion, minimizing the D-criterion leads to smaller prediction errors. For all of these reasons, we used the D-optimality criterion in our study.

The D-criterion value of a conjoint choice experiment with a design matrix X, collecting the attribute levels of the alternatives in each of the choice sets, is based on the so-called information matrix. The information matrix, which is inversely proportional to the variance-covariance matrix of the parameter estimators, is given by

$$I(X,\beta) = R\sum_{n=1}^{N} X'_n (P_n - p_n p'_n) X_n, \tag{4}$$

where $X_n=[x_{1n} \ldots x_{In}]'$, $p_n=[p_{1n} \ldots p_{In}]'$, and $P_n=\mathrm{diag}[p_{1n} \ldots p_{In}]$.

A D-optimal design maximizes the determinant of the information matrix, and consequently minimizes the determinant of the covariance matrix of the parameter estimates. An often-used measure in the marketing literature to express how good a design is in terms of the D-criterion is the D-error

$$D = \left\{\det I(X,\beta)^{-1}\right\}^{1/p}, \tag{5}$$

where p is the dimensionality of the parameter vector. Smaller values for the D-error indicate better designs.

In this paper, the design is constructed in a Bayesian fashion. Therefore, the uncertainty concerning the prior information about the parameter values is explicitly taken into account. If we denote the prior distribution for β by $f(\beta)$, the Bayesian D-error, denoted by D_B, is defined as the expectation of the D-error over the prior distribution of the parameter values:

$$D_B = E_\beta\left[\left\{\det I(X,\beta)^{-1}\right\}^{1/p}\right] = \int_{\Re^p} \left\{\det I(X,\beta)^{-1}\right\}^{1/p} f(\beta) d\beta. \quad (6)$$

The Bayesian D-optimal design is the one that minimizes the D_B-criterion. In practice, the D_B-value is approximated by drawing M random vectors β^m from the prior distribution $f(\beta)$ and computing

$$\widetilde{D}_B = \frac{1}{M}\sum_{m=1}^{M}\left\{\det I\left(X,\beta^m\right)^{-1}\right\}^{1/p} . \quad (7)$$

In our study, we used M=1000 draws.

2.3 Design Algorithm

The design construction algorithm used in this study is a modified Fedorov algorithm. The algorithm starts by building a candidate set, which is a list of all possible alternatives. A starting design with a specified number of alternatives I and choice sets N is constructed by randomly selecting the alternatives from the candidate set. Starting from the first choice set, every alternative in the starting design is then exchanged with every candidate alternative. For each exchange, a $\widetilde{D}_B$ –value is computed by using equation (7). An exchange is accepted if and only if it results in an improvement of the $\widetilde{D}_B$ –value. The first iteration is terminated once the algorithm has found the best exchanges for all alternatives in the starting design.

After that, the algorithm goes back to the first alternative and continues until no substantial improvement is possible any more. To avoid poor local optima, 100 different starting designs were generated and improved using the algorithm's exchange procedure in our study. Based on the efficiency criterion D_B-error, the design with the smallest $\widetilde{D}_B$ – value is considered the D_B-optimal design.

3. Comparing designs

In this section, we compare the interaction-effects design with the main-effects design in terms of the design efficiency and the minimal level overlap property. The design problem considered is the construction of a choice experiment with 11 choice sets of 2 alternatives described by 2 attributes. One attribute has three levels, while the other has only two levels. A short-hand notation for this problem is 3×2/2/11.

Effects-type coding is used throughout this paper. This means that, for an attribute with three levels, the vectors [1 0], [0 1], and [-1 -1] are assigned to the levels 1, 2, and 3, respectively. For an attribute with just two levels, -1 and 1 were used to code the two levels. As a prior parameter distribution, we considered the multivariate normal distribution with the p-dimensional identity matrix as a variance-covariance matrix, that is, $\beta \sim N(\beta_0, \mathrm{I}_p)$. For the main-effects design, we specified $\beta_0 = \beta_{0\mathrm{m}} = [-1\ 0\ -1]'$, so that the partworths for each of the attributes were evenly spaced between -1 and 1. For the 3-level attribute, a partworth of -1 corresponds to level 1, a partworth of 0 corresponds to level 2, and one of 1 corresponds to level 3. For the 2-level attribute, partworths of -1 and 1 correspond to the levels 1 and 2. It is thus assumed that the utilities increase with the levels of each attribute. In practice, it is often more difficult for a researcher to identify the relative importance of the interaction parameters in advance than it is for the main effects. Therefore, we assumed zero prior values for the interaction effects when specifying the prior parameters for constructing an optimal design for the interaction-effects model. So we used mean $\beta_0 = \beta_{0\text{int}} = [-1\ \ 0\ \ -1\ \ 0\ \ 0]'$ for the prior to construct the interaction-effects designs.

3.1 Evaluating design efficiency

The optimal designs for the interaction-effects model as well as the main-effects model are displayed in Table 2. The lower part of this table shows four $\widetilde{D}_B -$errors obtained for the four combinations of design and modeling strategies examined here. If the interaction-effects design is used to fit the main-effects model, then the $\widetilde{D}_B -$error is 0.731. However, the best design for that main-effects model has a $\widetilde{D}_B -$error of 0.679. The interaction-effects design is therefore (1-0.679/0.731)×100%=7.1% less efficient for estimating the main-effects model than the main-effects design. If we use the main-effects design to fit the interaction-effects model, the main-effects design is (1-0.876/1.154)×100%=24.1% less efficient for estimating the interaction-effects model than the interaction-effects design. It seems that using the main-effects design to fit the interaction-effects model results in larger efficiency losses than using the interaction-effects design to fit the main-effects model.

3.2 Evaluating level overlap

In the literature, minimal level overlap is considered as one of the properties which characterize efficient choice designs because, intuitively speaking, only the differences between attribute levels within a choice set are informative. In this subsection, we compare the degree of level overlap exhibited by the interaction-effects design with that exhibited by the main-effects design. From Table 2, it can be seen that in the interaction-effects design, the levels of attribute A and B are frequently repeated within a choice set. In the main-effects design, only attribute B does not satisfy the principle of minimal level overlap. The percentages of the cases in which the columns of the choice sets do overlap are 27.3% and 22.7% for the interaction-effects design and the main-effects design, respectively. It thus seems that the former design requires more level overlap than the latter.

Table 2

Optimal Design for the Main-effects and Interaction-effects Models

		Main-effects design		Interaction-effects design	
Choice set	Alternatives	A	B	A	B
1	I II	1 2	1 1	1 2	2 1
2	I II	1 3	2 1	3 1	2 2
3	I II	3 2	1 2	3 3	1 2
4	I II	3 1	2 2	3 1	2 1
5	I II	2 3	1 1	1 2	1 1
6	I II	2 1	2 1	2 3	2 1
7	I II	3 2	1 1	3 1	2 2
8	I II	2 1	2 2	1 2	2 2
9	I II	2 1	1 2	2 3	1 2
10	I II	1 2	2 1	2 3	1 1
11	I II	2 3	1 2	2 1	1 2
$\widetilde{D}_B-$error	Main-effects model	0.679		0.731	
	Interaction-effects model	1.154		0.876	

To further investigate the relationship between the number of interactions included in the model and the percentage overlap in the resulting design, we constructed another small design,

$2^3/2/11$, with three attributes acting at 2 levels in 11 choice sets with 2 alternatives each. Bayesian *D*-optimal design where constructed under the assumptions that one, two or three two-attribute interactions were present. The percentages of level overlap amount to 30.3%, 33.3% and 42.4% for the one-, two- and three-interaction designs, respectively. Thus, the more interaction terms are included in the design construction, the more level overlap is required.

Our results appear to be consistent with Zwerina et al. (1996), who mention that, in general, the presence of interaction effects necessitates overlap of attribute levels within choice sets to produce the contrasts necessary to estimate them, and with Chrzan et al.(2000), who conclude that designs with little level overlap within choice sets are good at measuring main effects, while designs with a lot of level overlap better for measuring higher-order effects.

4. Evaluating prediction accuracy

The purpose of this section is to evaluate the predictive performance of the interaction-effects design relative to the main-effects design, and of the interaction-effects model relative to the main-effects model. The goal is to find design and model strategies that, when combined together, provide an approach that is robust against misspecification of the model.

4.1 **Measure for prediction accuracy**

In this paper, we use the expected root mean-squared error of the predicted probabilities, $ERMSE_P$, as a measure for prediction accuracy. It is obtained by comparing the predicted and true probabilities:

(8) $$ERMSE_P = \int_{\Re^p} \left[\left(P(\hat{\beta}) - P(\beta^*)\right)' \left(P(\hat{\beta}) - P(\beta^*)\right) \right]^{1/2} \pi(\hat{\beta}) d\hat{\beta},$$

where $\pi(\hat{\beta})$ is the distribution of the estimates, $P(\hat{\beta})$ is the vector containing the predicted probabilities computed using the estimated parameters $\hat{\beta}$, and $P(\beta^*)$ is the vector of the true probabilities. As in Kessels et al.(2005), the $ERMSE_P$ was computed from the set of all possible choice sets of size two. Since the design problem considered here has 3×2=6 different alternatives, there are $\binom{6}{2}$=15 possible choice sets.

From the definition, it is clear that the $ERMSE_P$-values are inversely related to the predictive performance: the smaller the $ERMSE_P$-value, the better the predictive performance. In this work, the $ERMSE_P$-value is approximated by

(9) $$ERMSE_{\hat{P}} = \frac{1}{T} \sum_{t=1}^{T} \left[\left(P(\hat{\beta}^t) - P(\beta^*)\right)' \left(P(\hat{\beta}^t) - P(\beta^*)\right) \right]^{1/2},$$

where T represents the number of simulations. In our study, we used T=500 as this turns out to be sufficient to obtain reliable results.

4.2 Simulation study

Two basic stages are considered in this study, namely, the design stage and the estimation stage. The interaction terms can be incorporated into both stages, one of two stages or neither of two stages. In our simulation study, we evaluated different combinations under several true situations. Each true situation corresponds to different sizes of the interaction effects.

The designs used in this simulation study are those of Table 2. Responses were generated based on some true parameters β^*, which were randomly sampled from $\beta^* = \beta_0 + \nu\sigma$, ν~

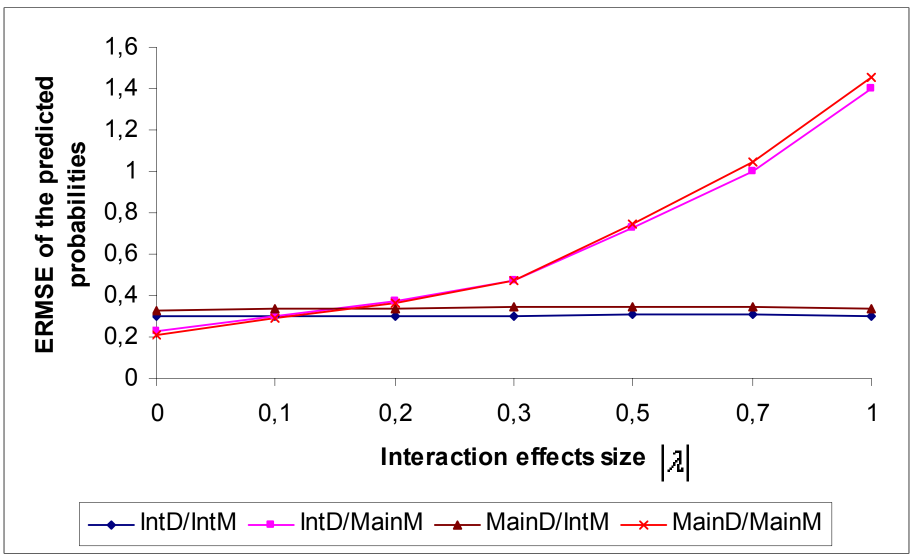
$N(0,\mathrm{I}_p)$, where as before I_p is the p-dimensional identity variance matrix, σ=0.1, $\beta_0 = \beta_{0\mathrm{m}}$=[-1 0 -1]′ for situations where we assume that only main effects exist and $\beta_0 = \beta_{0\mathrm{int}}$=[-1 0 -1 λ 0.5λ]′ for situations where we assume that interaction effects exist. The parameter λ was used to manipulate the magnitude of the interaction effects in the study. The larger the absolute values of λ, the larger the interaction effects in the true situation. In our study, we consider the following values for λ: -0.1, -0.2, -0.3, -0.5, -0.7 and -1. Together with the situation in which no interaction effects are assumed (which comes down to assuming λ =0), this means that we considered seven different situations in total. For each situation, we considered both the interaction-effects model and the main-effects model and drew 100 true parameter values β^*. For each draw, we simulated T=500 times responses of R=22 respondents and computed the corresponding predicted probabilities. The $ERMSE_{\hat{P}}$-value is then calculated using Equation (9). Finally, we use the average $ERMSE_{\hat{P}}$ over the 100 draws to represent the predictive performance of each model for each situation. Note that the responses are simulated as follows: for each choice set n and each respondent r, a random number is drawn from the uniform distribution U[0,1], and that random number is compared to the true logit probability of choosing the first alternative in the choice set. If the random number is smaller than or equal to the probability, the response variables y_{1nr} and y_{2nr} are set to one and zero, respectively. In the opposite case, y_{1nr} and y_{2nr} are set to zero and one, respectively.

4.3 Predictive performance comparison of the two design options

The relative performance of the interaction-effects design and the main-effects design are visualized in Figure 1 for different situations. The magnitudes of the attribute interactions $|\lambda|$ are shown on the horizontal axis, and the $ERMSE_{\hat{P}}$, reflecting the predictive performance of the underlying models, is shown on the vertical axis. Note that $|\lambda|=0$ corresponds to the situation where the model that generates the responses contains no interactions. The predictive performance of the interaction-effects model and the main-effects model are represented by "IntD/IntM" and "IntD/MainM" when the interaction-effects design is used to fit both models, and by "MainD/IntM" and "MainD/MainM" when the main-effects design is used to fit both models.

Figure 1

Comparison between Designs and Model in Terms of Predictive Performance



When we use both the interaction-effects design and the main-effects design to fit the interaction-effects model, the comparison between "IntD/IntM" and "MainD/IntM" shows that the former design tends to perform better than the latter even when the interaction effects are small. The small gains in efficiency of the former design over the latter design to fit an interaction model are not very sensitive to the size of the interaction effects.

If we use both designs to fit the main-effects model, the comparison between “IntD/MainM” and “MainD/MainM” shows that, when no interactions or small interactions are present, the main-effects design tends to perform slightly better. However, when the interaction effects are reasonably large, the interaction-effects design performs a little bit better than the main-effects design.

Given the small differences observed between the two design options for a given models, we can conclude that the choice of a design strategy is not critical. If large interaction effects are suspected, there is a small edge in favour of the interaction-effects design.

4.4 Predictive performance comparison of the two model options

Our goal here is to select the best model (either the interaction-effects model or the main-effects model) in the estimation stage when there is uncertainty about the presence of important interaction effects. For that purpose, we now turn our attention to the predictive performance of the two models.

The curves in Figure 1 clearly indicate that, when no or small interactions are present, the main-effects model performs slightly better than the interaction-effects model. This is true for both the situation in which the main-effects design is used and the situation in which the interaction-effects design is used. This is not unexpected, because for cases in which the interactions are very small the gains from incorporating interactions can not offset the costs from estimating more parameters. However, if large interaction effects are present, the interaction-effects model substantially outperforms the main-effects model. Evidently, the larger the sizes of the interaction effects, the worse the predictive performance of the main-effects model compared with the interaction-effects model.

In addition, an important conclusion that can be drawn from Figure 1 is that for the interaction-effects model "IntD/IntM", the predictive performance of the interaction-effects model does not change very much as the true situation changes from zero interaction effects to large ones: the $ERMSE_{\hat{p}}$-values range only from 0.301 to 0.306. However, this is not the case for the main-effects model. The predictive performance of the main-effects model largely depends on the magnitude of the interaction effects: the $ERMSE_{\hat{p}}$-values range from 0.230 to 1.398.

The conclusion of this all is that it is substantially safer in terms of predictive performance to estimate an interaction-effects model than the main-effects model when there is some doubt about the true nature of the model. Given the slight edge in favour of using the interaction-effects design, combining the interaction-effects design with the interaction-effects model is thus certainly a robust strategy to adopt. In the next section, we study the sensitivity of this result to the prior distribution used in the design stage for the parameter vector β.

4.5 Sensitivity analysis

4.5.1 Prior parameters of the interaction terms

In the simulation study so far, we used an optimal interaction-effects design constructed using the specific prior $\beta_0 = \beta_{0\text{int}}$=[-1 0 -1 0 0]′ and concluded that combining it with the interaction-effects model in the estimation stage protects the researcher against active interaction effects.

However, different prior parameters lead to different optimal designs. It is therefore of interest to know whether the results obtained are sensitive to the choice of prior parameters used to construct the interaction-effects designs. To investigate this, we constructed two new designs with different sizes for the prior parameter values for the interaction effects: $\beta_0 = \beta_{0\text{int}}$=[-1 0 -1 -1 0]′, and $\beta_0 = \beta_{0\text{int}}$=[-1 0 -1 -1 -0.5]′, respectively. We keep the prior parameters of the main effects terms constant and change only the prior parameter values of the interaction terms. This makes sense because it will usually be harder for researchers to produce good prior guesses of the magnitude of the interaction effects.

The plots obtained from the two new designs exhibit similar patterns to those in Figure 1 and are therefore not shown here. The results thus confirm the robustness of the combination of the interaction-effects design with the interaction-effects model.

4.5.2 Sign of the prior parameters of the interaction terms

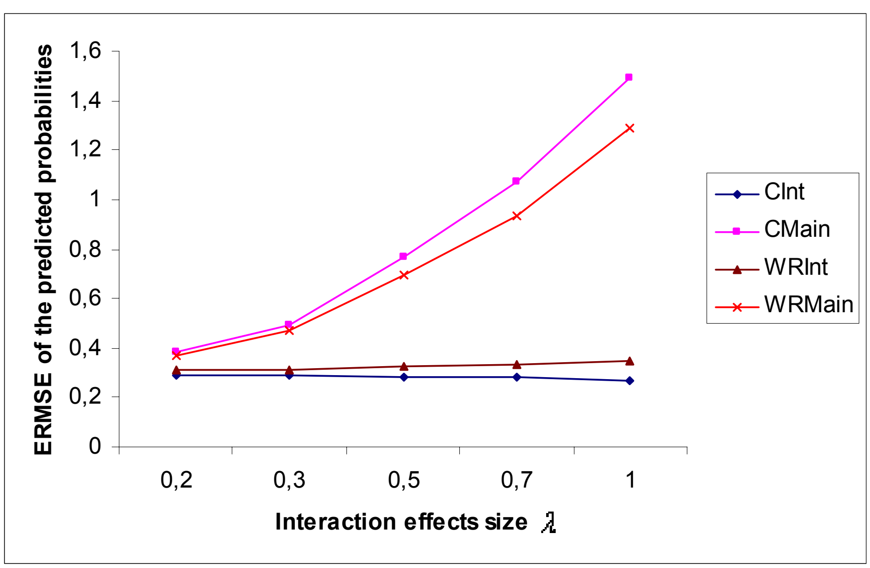
The purpose is to examine whether the robustness still holds if the prior values for the interaction effects specified by the researcher when constructing an interaction-effects design are totally wrong. To investigate this, we conducted an additional simulation study with the design produced by $\beta_0 = \beta_{0\text{int}}$=[-1 0 -1 -1 -0.5]′ and with data generated from models with opposite signs for interaction-effects: $\beta_0 = \beta_{0\text{int}}$=[-1 0 -1 λ 0.5λ]′, where λ is positive. The predictive performances of the interaction-effects model and the main-effects model under the situation where the prior parameters provide wrong information are denoted by “WRInt” and “WRMain”, respectively, and those obtained in the situation where the prior parameters provide correct information are represented by “CInt” and “CMain”, respectively. The results are shown in Figure 2.

Figure 2 shows that incorrect prior information regarding the interaction effects has a negative impact on the quality of the predictions. However, the increase in $ERMSE_{\hat{p}}$ because of incorrect prior information is much smaller than the impact of not using the interaction-effects model in the estimation stage. The figure clearly shows that the benefit of using the interaction-effects model is considerably larger when correct prior information is utilized during the design construction. However, even when the prior information is completely wrong, using the interaction-effects design combined with the interaction-effects model is still the most robust thing to do.

In summary, the simulation studies in this section show that the benefits of using the interaction-effects model in the design and analysis stages are not very sensitive to the sizes of the prior parameters for the interaction terms used to construct the interaction-effects designs, nor to the signs of the prior parameters for the interaction effects.

Figure 2

Predictive Performances of the Interaction-effects Designs Obtained Under Correct and Incorrect Information Regarding the Interaction Effects



5 Conclusion

In this paper, we studied the predictive performances of two types of designs and two types of models with respect to the predictive accuracy. The studies led to the practical recommendation that, in situations where a researcher is not sure whether or not interaction effects exist, incorporating interactions into both design stage and estimation stage is the most robust strategy as it will usually provide the best predictive accuracy. The sensitivity studies show that the recommended robust strategy is not sensitive to the prior information of the interaction terms. So, even if the researcher has no idea about the magnitude of the interaction effects, it does not harm to use a Bayesian *D*-optimal interaction-effects design.

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